Global static and dynamic car body stiffness based on a single experimental modal analysis test

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Abstract
Global car body stiffness is an important design attribute in vehicle design. Therefore accurate characterization of this stiffness is needed. The current industrial method for static stiffness determination has several downsides, amongst others its time consuming set-up preparation. Dynamic characterization by means of modal analysis on the other hand has high repeatability and a less time consuming set-up preparation. This paper describes and experimentally validates the industrial implementation of a method to overcome the downsides of the classic static test by determining both global static, as well as dynamic stiffness based on a single modal analysis test. The method combines several techniques available in literature. Theory behind the method is elaborated, simulation tests show the potential of the method and finally experimental validation on a full body-in-white proves industrial applicability of the proposed technique.

1 Introduction

Global car body stiffness is an important design attribute in vehicle design. Accurate body-in-white structural identification, including global static stiffness identification is therefore of high importance. Increasingly CAE techniques are used in this regard. Nevertheless experimental car body structural identification is needed to verify and update structural finite element models. In automotive industry different tests are performed, ranging from static deformation tests, experimental modal analysis to operational testing on laboratory test benches and the road.

Global dynamic stiffness characterization is an elementary part of this and is determined by an experimental modal analysis test. These dynamic tests are used for target verification, troubleshooting and finite element model updating. For body-in-white testing, measurements are performed under so-called free-free boundary conditions, which means that the car body is decoupled from the environment. The practical realization of this condition is well defined and realized by hanging the structure or mounting it on very soft springs.[16] Main advantages of this type of testing are the good consistency with which these free-free boundary conditions can be realized and the relatively low influence of small changes in the test set-up, resulting in high repeatability.

The global static stiffness is measured on a static deformation test bench. Different load cases are available. This work will be limited to global static bending and global static torsional stiffness determination. During a global bending test, forces are applied at the front seat locations, while the body is constrained at front and rear shock towers, as shown in figure 1. The static bending stiffness results from the ratio of the applied load to the maximum deflection along the rocker panel and tunnel beams.

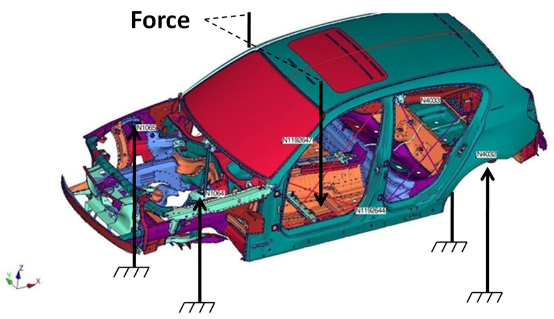

Figure 1: Global static bending test

For global static torsion stiffness, a static moment is applied to the body-in-white at the front shock towers, whereas the rear shock towers are constrained, as shown in figure 2. The torsion angle is defined as the resulting deformation angle between the front and rear shock towers. The corresponding torsion stiffness can be calculated as the ratio of applied static moment to the torsion angle.

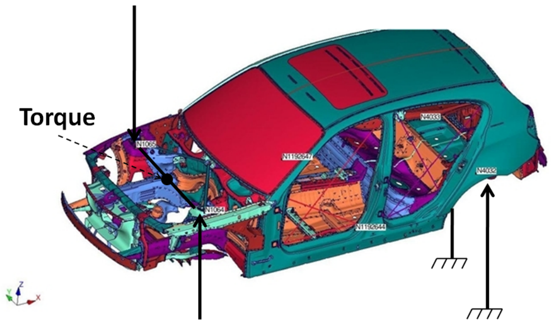

Figure 2: Global static torsion test

There are some concerns with regard to the current static testing method. First concern is, that fixturing schemes are as plentiful and varied as the car manufacturers that design and fabricate them.[7] Both statically as well as non-statically determined set-ups are possible. For the bending test, a possibility to avoid over-constraining the body-in-white is supporting it at the two front shock towers in z-direction, constraining it in x and z-direction at one rear shock tower and in x,y and z-direction at the other. A possibility to realize the torsion set-up boundary conditions without over-constraining the body, is by constraining it at one rear shock tower in x- and z-direction and at the other in x-,y-,z-direction. Static stiffness values will depend strongly on these boundary conditions and are therefore not easy to compare.

Second concern is, that force application can differ as well. For a bending test, there are different possibilities to apply the bending loads. Bending loads can be applied as line forces with a bending beam or more local by means of an hydraulic cylinder, making results from both difficult to compare. Torsion torque can be applied with or without torsion beam.

Third concern is, that in practice it is not possible to realize certain boundary conditions in a perfect way.

No clamping device is infinitively stiff, which implies, that the stiffness of the clamping device can be of influence on the result of the overall stiffness measurement. In the current trend of increasing body-in-white stiffness, realizing reliable clamping devises becomes more and more important. Current values of body-in-white stiffness at BMW are well above the 5.2kN/mm for the static bending stiffness and 8.5kNm/deg for static torsion stiffness indicated in [2], which implies clamping device stiffness needs to exceed this value significantly imposing strong demands on clamping device design. If the assumption is made, that clamping devices should be at least ten times stiffer than the measured object; high demands are set to the clamping device construction in order not to influence the test result.

A last concern, which is highly relevant in industry, global overhead of a classic static test is quite significant. Test set-up construction and sensor calibration are very time consuming, resulting in high test costs.

These concerns create an opportunity to look for an alternative testing method without these disadvantages. The employment of a modal analysis based technique should be possible, because the dynamic and static behavior of a structure are linked. Moreover, a method based on an experimental modal analysis test under free-free boundary conditions would have the extra added value of the high repeatability of the realization of the free-free boundary conditions. This overcomes the first concern of the current testing technique. Furthermore significant measurement efficiency improvement can be realized, since a modal analysis test is a standard procedure within automotive structural identification. Therefore it is possible to use the same set-up for both tests reducing overhead costs drastically and assuring consistent set-up for static and dynamic stiffness estimation.

For the bending test the six bending base points, indicated by the arrows in figure 3, are needed in order to be able to apply forces and boundary conditions. Moreover, since bending stiffness results from maximum deformation at the rocker panels, displacements need to be known at several locations along the rocker panels. A possibility is the use of the locations market by the dots on the rocker panels. The full set of locations will be referred to as the bending points. If more base points are included it is possible to point out the displacement and location of the maximum bending deformation along the rocker panels. Further in this paper however only six base points will be used for the bending load case in order to keep complexity limited when illustrating the method. In case of the torsion test the four significant torsion base points are indicated by the arrows pointing up. At least in these points displacements should be known to calculate respectively global bending and torsion stiffness. Therefore, for a modal analysis based technique, modal information should be available at least at these points.

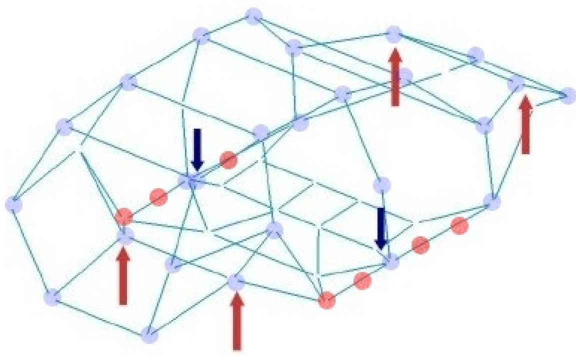

Figure 3: Equivalent geometry of structure

2 Methods for global static stiffness deduction from dynamic measurements - General overview

The approaches to estimate static stiffness by means of dynamic information can traditionally be divided in two categories.

2.1 Quasi-static operational dynamic measurements

The techniques of the first category excite the investigated structure at a low frequency and measure displacements. Boundary conditions need to be applied, when performing the excitation. The resulting semi-static stiffness value approximates the static stiffness. De Meester[3] applied this technique successfully for real-time optical deformation measurements of flexible manipulators. Main advantage of this technique is the relatively low measurement effort. The insurmountable problem of this category of approaches however lies in the fact, that the boundary conditions of the dynamic test set-up should be exactly the same as the ones used in the equivalent static test. For a body-in-white static test, this implies that the full static set-up would still have to be built up in exactly the same way as it is done today. So for this application this would not result in any time gain compared to the present static testing method. Therefore our suggested statics from dynamics method will not be based on this approach.

2.2 Static stiffness deduction based on free-free structural dynamic identification measurements

The methods of the second category on the other hand are based on compliance data of the investigated structure. The compliance matrix H relates displacements to forces for all measured frequencies.

$$\left\{ \; X \; \right\} = \left[\; H_{i,j} \; \right] . \left\{ \; F \; \right\} \tag{1}$$

Catbas, Allemang et al. [4][5][6] use compliance information to estimate damage on a bridge due to stiffness changes. Rediers [7] suggested the use of free-free frequency response function (FRF) measurements to determine the static stiffness of a frame structure. Reichelt[8] implemented the Rediers approach for a body-in-white for both statically as non-statically determined systems. A technique of relating vehicle structural modes to stiffness as determined in static determinate tests by D.Griffiths[9]. This paper indicates the link between the static and dynamic stiffness of a vehicle. The presented statics from dynamics technique is based on the findings of Reichelt and uses free-free compliance data of the structure as input for the static stiffness estimation. Moreover the statics from dynamics technique will be able to indicate the modal contribution of the different flexible modes to the global static torsion and bending stiffness.

Input is compliance data in respectively the four torsion base points in case of a torsion test and the bending points in case of a bending test. Rediers et al.[7] include only translations in Z-direction in their technique. Based on the good results of the Rediers apporach for frame constructions, the statics from dynamics method also limited to Z-direction displacements in the presented test cases. Nevertheless the proposed theory is also applicable for three degrees of freedom (DOF). In theory six degrees of freedom is possible. However measurement complexity will increase significantly. Therefore a four by four compliance matrix forms the input for the torsion calculation, whereas a six by six compliance matrix is used for the bending calculation.

There are two parallel tracks to determine the compliance matrix:

2.2.1 Compliance matrix estimation at 0Hz

Modal synthesis based compliance matrix estimation
This technique uses the modal synthesis technique to construct the compliance matrix based on measured data. Main advantages are that compliance values in a region at about 0Hz can be calculated directly. Furthermore not all points should be excited to fully characterize the compliance matrix. Moreover, the modal contribution of each mode to the modal model and consequently to the global static stiffness can be quantified. Main downside however is the need for a full modal model including residual stiffness to cope with modal truncation. The latter is not straightforward and demands high quality measurements. This technique will be further elaborated in section 3

The full experimental determination of the compliance matrix
This track is based on a full direct measurement of the compliance matrix. All elements of the compliance matrix are determined directly by means of frequency response function measurements. However compliance data should be determined at very low frequency. Since in an experimental test set-up it is not possible to fully achieve free-free conditions, the suspension constraints the rigid body degrees of freedom and therefore will shift the rigid body modes from zero Hz to low frequencies.[10] Moreover accuracy of dynamic accelerometers is lower at low frequency. Therefore making it impossible to use measured compliance data from a frequency range close to 0Hz. This can be overcome by using the frequency response functions measured for frequencies well above the eigenfrequencies corresponding to these shifted rigid body modes and extrapolating these to zero Hz to derive the compliance matrix at 0Hz. Main advantage of this technique is the possibility of determining the compliance data without the need for complex compensation techniques for residual components. Main downside however is the need for an accurate extrapolation approach, as it will determine the quality of the estimated compliance matrix. This technique will be elaborated further in section 3.4.

2.2.2 Virtual static test

The result of the previous steps is the two-dimensional compliance matrix of equation 1 linking forces in z-direction to z-translations.

On this matrix equation a virtual static test is performed by applying static boundary conditions. There are two possibilities for applying the static boundary conditions, the use of which is determined by whether the system is statically determined or not.

Equivalent forces method[7]
If the system is statically determined, statically determined reaction forces are only dependent on geometry information and therefore relatively easy to calculate and can be used as force matrix in equation 2.2. For example in case of bending, reaction forces shown in figure 4 can be derived from force equilibrium:

$$\begin{aligned} R_{Bend,Rr}.A &= F_{Bend}.B \\ R_{Bend,Fr}.A &= F_{Bend}.(A-B) \end{aligned} \tag{2}$$

Solving these equations leads to:

$$R_{Bend,Fr} = \frac{A-B}{2.A}.F_{Bend} \tag{3}$$

$$R_{Bend,Rr} = \frac{B}{2.A}.F_{Bend} \tag{4}$$

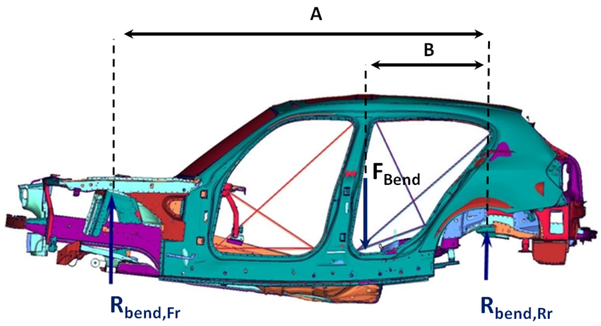

Figure 4: Reaction forces in a global static bending test

In case of torsion due to applied moment M_{appl}, reaction forces R_{Tors} shown in figure 5 result from the torque equilibrium

$$M_{appl} = R_{Tors,Fr}.C \tag{5}$$
$$R_{Tors,Fr}.C = R_{Tors,Rr}.D \tag{6}$$

Therefore:

$$R_{Tors,Fr} = M_{appl}/C \tag{7}$$
$$R_{Tors,Rr} = M_{appl}/D \tag{8}$$

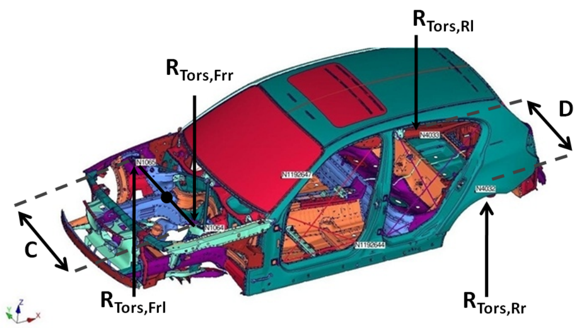

Figure 5: Reaction forces in a global static torsion test

Prescribed displacements method

For non-statically determined systems, reaction forces are not straightforward to calculate. Therefore a different approach to do virtual static testing is suggested. In contrast to the equivalent forces method, no loads and reaction forces are used, but loads and prescribed displacements are set in matrix equation 1. Prescribed displacements are set in the displacements matrix of equation 1. For example in a hinge, displacement in the displacements matrix of equation 1 is set to zero in Z-direction at the corresponding base point. Applied forces are set in the force matrix of equation 1. In all points, where displacement is not known and no external force is applied, external force is set to zero in the force matrix of equation 1. This approach, which is valid both for statically determined as for non-statically determined systems, will be referred to as the prescribed displacements method. For the bending test shown in figure 4 six base points were measured and therefore transforming matrix equation 1 to:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ X_5 \\ X_6 \end{bmatrix} = \begin{bmatrix} H_{1,1} & H_{1,2} & H_{1,3} & H_{1,4} & H_{1,5} & H_{1,6} \\ H_{2,1} & H_{2,2} & H_{2,3} & H_{2,4} & H_{2,5} & H_{2,6} \\ H_{3,1} & H_{3,2} & H_{3,3} & H_{3,4} & H_{3,5} & H_{3,6} \\ H_{4,1} & H_{4,2} & H_{4,3} & H_{4,4} & H_{4,5} & H_{4,6} \\ H_{5,1} & H_{5,2} & H_{5,3} & H_{5,4} & H_{5,5} & H_{5,6} \\ H_{6,1} & H_{6,2} & H_{6,3} & H_{6,4} & H_{6,5} & H_{6,6} \end{bmatrix} . \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_{Bend} \\ F_{Bend} \end{bmatrix} \tag{9}$$

Row one to four represent the data at the four shock tower points and row five to six at the force application points. For the torsion test, a similar approach is used.

Finally the system of equations should be solved, resulting in knowledge of all displacements and forces acting on the structure.

2.2.3 Postprocessing

The postprocessing step consists of stiffness value calculation using the forces and displacements determined in the previous step.

Equivalent forces method
For the bending test, total deflection at one side of the body-in-white can be calculated by adding the amplitudes of the displacements at the front and rear schock towers due to reaction forces $R_{Bend,Fr}$ and $R_{Bend,Rr}$ to the displacement at the force application location. The global static bending stiffness is the ratio of the applied load to this maximum deflection along the rocker panel. For the torsion test a total torsion angle difference is calculated, as the sum of the torsion angle at the front schock towers due to the applied static torque and the torsion angle at the rear shock towers due to the reaction forces $R_{Tors,Rr}$. The ratio of the applied static torque to the total torsion angle difference defines the global static torsion stiffness.

Prescribed displacements method
The global static bending stiffness results from the ratio of the applied load to the maximum deflection along the rocker panel. The torsion angle is defined as the resulting deformation angle between the front and rear shock towers. The corresponding torsion stiffness can be calculated as the ratio of applied static moment to the torsion angle.

3 Modal synthesis based compliance matrix estimation[16]

The suggested method to determine the global static stiffness is based on the compliance data of the structure. This section discusses the modal synthesis technique to construct the compliance matrix based on measured data. Since the quality of the estimated static stiffness is dependent on the quality of the used compliance matrix, accurate compliance matrix estimation is highly important. The quality of the compliance matrix consisting of synthesized FRFs is dependent on the quality of the underlying estimated modal model of the structure. The compliance matrix H can be approximated by:

$$[H(j\omega)]_{N_o \times N_i} = \frac{[V]_{N_o \times 2N_m}}{[j\omega\,[I] - [\Lambda]]_{2N_m \times 2N_m}} . [L]_{2N_m \times N_i} + [UR]_{N_o \times N_i} - \frac{[LR]_{N_o \times N_i}}{\omega^2} \tag{10}$$

where:

- $[V]$: modal vector matrix
- $[\Lambda]$: diagonal matrix with system poles
- $[L]$: modal participation factor matrix
- $[UR]$: upper residual term matrix
- $[LR]$: lower residual term matrix
- N_o: Number of output stations
- N_i: Number of input stations
- N_m: Number of observable modes

A good modal model finds a good balance between the following three components:

- Rigid body modes
- Flexible modes
- Compensation terms

3.1 Rigid body modes

The rigid body modes characterize the rigid motion of the structure. Experimental determination of these modes is a possibility. However, a second and better method is constructing the rigid body modes using mass, center of gravity and inertia properties of the structure as calculations inputs. The center of gravity and the inertia properties can be calculated from FRF-measurements of the structure. A range of techniques to do this were already published, amongst others by Bianchi et al.[11], Bretl et al.[12] and Reichelt[8].

3.2 Flexible modes

Secondly, the flexible modes of the structure have to be determined by means of a modal analysis in a selected frequency band. First term in equation 10 results from this mode category. Time domain methods and frequency domain methods such as the Polymax method[13] can be used to determine eigenfrequencies and corresponding mode shapes. Modepicking should be performed with care since this is a determining factor for the quality of the modal set. Furthermore the quality of the measurements is highly important. Special care should be given to the physical realization of the free-free conditions, whereas an appropriate number of driving point measurements should be included to assure modal model estimation of high quality, especially with regard to scaling.

3.3 Residual compensation

The last contributions are residual compensation terms that account for the influence of modes outside the simulated/measured frequency band. To overcome this modal truncation, two approaches are used. For finite element simulations the modal set is completed by means of residual vectors. In case of experimental measurements, upper and lower residuals are discussed.

Residual vectors [14][15]

Residual vectors are frequently used in the component mode synthesis (CMS) model reduction technique. Flexible modes have only been determined in a limited frequency band. Although most of the dynamic structure response may be captured, the predicted system response may not be fully accurate. This is due to modal truncation of higher frequency modes. When focussing on the statics, the neglected dynamic modes also contribute statically to the total response. Therefore the static system response may be inaccurate. This modal truncation problem is overcome by the computation of residual vectors, that account for static contribution of the modes not included in the used modal base. A way to compute residual vectors is to compute attachment modes, which are determined starting from the static response of the structure to unit forces at the boundary condition and input force degrees of freedom. These attachment modes are orthonormalized with the modal model resulting in one mode set.

Upper residuals[16]
When experimental data is used as input, the influence of modes outside the measured frequency band can be approximated by means of upper and lower residual terms. In case of proportional damping the influence of modes above the measured frequency range can be approximated by a real constant (upper residual), whereas the effect of modes below the measured frequency range also by a constant (lower residual) divided by $-\omega^2$, as indicated in equation 10. Since the rigid body modes are included in the modal model and flexible modes are determined in a frequency range starting at a frequency as close to the rigid body modes as possible all contributions at the low frequency side of the used frequency band should be accounted for. Therefore only the upper residual term matrix $[UR]_{N_o \times N_i}$ is appropriate. The upper residual is used to estimate a residual stiffness for each of the base point degrees of freedom where the body will be constrained during the virtual static test, discussed in Section 2.2.2. This residual stiffness will be included in the virtual static test by means of discrete springs. Instead of constraining the body rigidly to the ground springs representing the residual stiffness are placed in between the body and the ground during the virtual static test. For both the torsion and bending test springs are placed at the four base points at the front and rear shock tower locations.

3.4 Full experimental determination of the compliance matrix[19]

A second approach to estimate the compliance matrix is direct experimental determination. Main advantage is that there is no need for determination of the residual compensation. Hammer excitation combined with acceleration measurements are used to determine all the frequency response functions (FRF's) between all base points of the structure. These measured FRF's are the elements of the modal matrix. The modal matrix can subsequently be transformed to the compliance matrix of equation (1) by division by $-\omega^2$.

Since in general measured responses below 5Hz are not useful, due to the presence of rigid body modes and lower sensor accuracy in this frequency range, a curve fit is necessary to determine compliance matrix values at low frequency. Each frequency response function of the compliance matrix is fitted at low frequency, as if it were an undamped single degree of freedom spring-damper system. Such an undamped single degree of freedom spring damper system is characterized by

$$m\ddot{x}(f) + k.x(f) = F(f) \tag{11}$$

with:

- m: the mass
- k: the stiffness
- F(f): the applied force corresponding to frequency f
- x(f): the displacement corresponding to frequency f

After Fourier transformation this equation yields:

$$(k - \omega^2.m) = \frac{F(f)}{x(f)} \tag{12}$$

Transformation from pulsations to frequencies by means of $\omega = 2\pi.f$ results in the following equation:

$$(k - (2\pi f)^2.m) = \frac{F(f)}{x(f)} \tag{13}$$

In a first step *m* and *k* are estimated from that part of the measured FRF's where measurement quality is high using equation 13. In this equation the applied load divided by the displacement is known for each frequency f from the part of the FRF measurement where the quality is high resulting in knowledge of the parameters *m* and *k*. The resulting *m* and *k* values are filled in in equation 13 and used to estimate the $\frac{F(f)}{x(f)}$ value for the frequencies where the quality of the FRf measurement was low.
The fitted compliance matrix is used as input for the virtual static test.

4 Modal contribution of flexible modes to global static stiffness

As indicated in section 2.2.1, one of the main advantages of the synthesis based compliance matrix estimation is, that the modal contribution of each mode to the modal model of the system and consequently to the global static stiffness can be quantified. This allows the designer to focus on modes that contribute significantly. Since most load cases that act on the car consist of dynamic and static contributions, significant design improvement can be achieved by establishing a good tuning of the different modes using body stiffness modifications, since it will ameliorate both static as well as dynamic behavior of the car.
The definition of the synthesized compliance matrix for proportionally damped systems[18] is expressed by:

$$[H_{syn}(j\omega_k)] = \sum_{i=1}^{N} \frac{\{\phi\}_i \{\phi\}_i^T}{(\omega_{n_i}^2 - \omega_k^2) + j2\varepsilon_i \omega_{n_i} \omega_k} \tag{14}$$

with:

- N: the number of included modes
- $\{\phi\}_i$: the *i*th mass normalized mode shape
- ω_{n_i}: the *i*th natural frequency
- ε_i: the *i*th modal damping ratio
- ω_k: the *k*th frequency

The N $\{\phi\}_i$ vectors consist of the different components of the modal model included in the modal synthesis. This facilitates the possibility to estimate the modal contribution of each mode shape to the compliance matrix at low frequency and consequently to the overall static stiffness of the system. The contribution of the different modes can be visualized by stiffness contribution graphs. The abscissa of these graphs lists the number of dynamic modes included in the modal synthesis, whereas the ordinate shows the corresponding static stiffness value. By including the dynamic modes one by one in the modal synthesis, the contributions of the different modes become visible in the stiffness contribution graph. Examples of such graphs are shown in section 5.

5 Method verification using FE models

In order to verify the proposed method, tests were conducted. A Finite element (FE) test was used to prove, that the suggested method works theoretically. Residual vectors were included in the modal model. Since by means of the residual vectors the modal model should be complete, stiffness estimation based on the suggested method should be highly accurate. FE tests were based on following approach: first, a free-free simulation of the investigated structure was performed to determine the structure's modal model. All three contributions of the modal model: rigid body modes, flexible modes and residual vectors in the six base points were simulated. This FE modal model was used as input information for the static stiffness calculation used to prove the method. Therefore the modal synthesis based compliance matrix estimation will be used.

Several structures were the subject of our research. The general conclusions of these validation simulations will be presented.

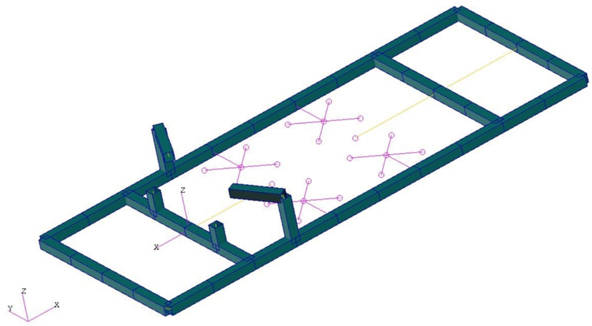

Figure 6: Frame construction

5.1 The Frame construction

A first important finding is the significant influence of the number of flexible modes taken into account in the synthesis. For example for the frame construction, shown in figure 6, the influence of the residual vectors to reach the correct static stiffness value was significantly lower in the case that more flexible modes were used. In each of the subsequent cases, shown in figures 7, 8 and 9, more flexible modes were taken into account and the influence of the residual vectors became less important (grey zone in graph). This underlines the assumption, that enough flexible modes need to be taken into account to minimize modal truncation effects. Otherwise quite some information is still gathered in the vectors. These residual vectors characterize the residual flexibility and compensate for that part of the flexibility that was not accounted for by flexible modes. The flexibility will therefore be underestimated. This implies that the stiffness, which is the inverse of the flexibility, will be overestimated. Figures 7, 8 and 9 illustrate this conclusion. The more flexible modes are taken into account, the more the stiffness value will decrease to the right value and the less residual vectors will be of influence. Therefore flexible modes should be included until stabilization in stiffness value occurs.

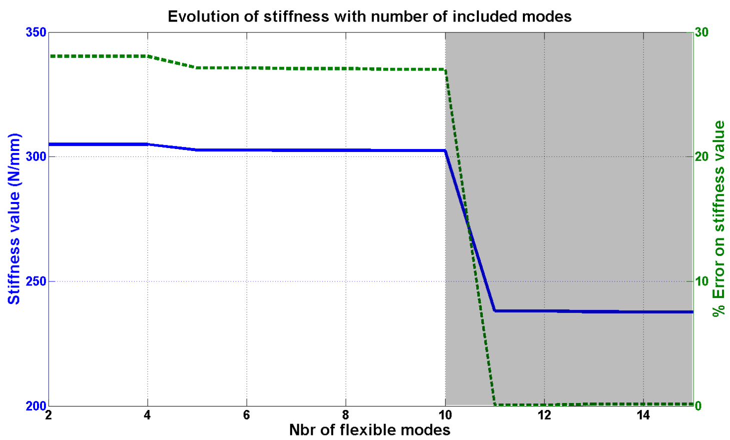

Figure 7: Evolution of the stiffness value with the number of modes taken into account (-) and the evolution of the error of the stiffness value with respect to the real value(- -). In total 10 flexible modes are taken into account. Residual vectors indicated by grey zone

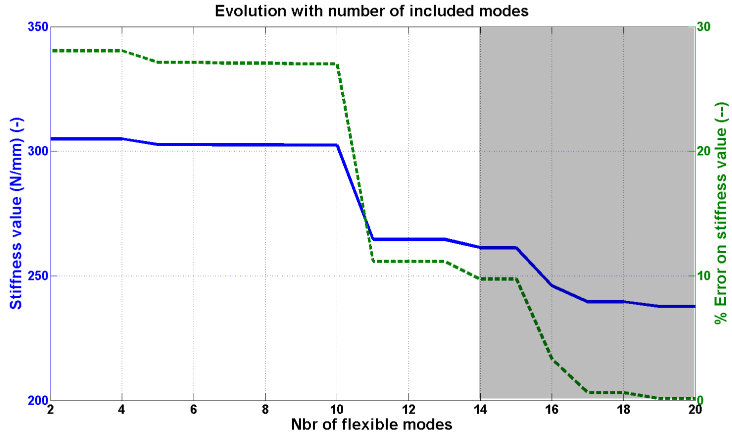

Figure 8: Evolution of the stiffness value with the number of modes taken into account (-) and the evolution of the error of the stiffness value with respect to the real value (- -). In total 14 flexible modes are taken into account. Residual vectors indicated by grey zone

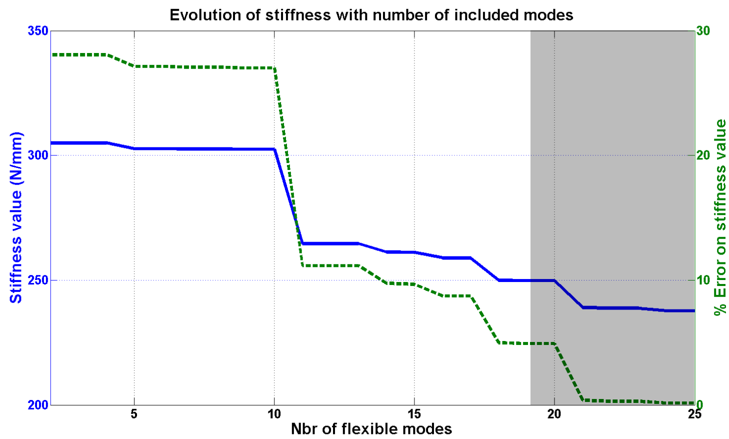

Figure 9: Evolution of the stiffness value with the number of modes taken into account (-) and the evolution of the error of the stiffness value with respect to the real value (- -). In total 19 flexible modes are taken into account. Residual vectors indicated by grey zone

5.2 Body-in-white

For the body-in-white, a modal model was simulated using FE. This model consisted of rigid body modes, flexible modes and residual vectors. In order to have a good insight in the evolution of the found stiffness value with number of included modes, a large number of flexible modes was included in the modal model, 61 to be exact. The modal model was used as input for the statics from dynamics program to calculate static bending stiffness value. The locations of forces and supports of the equivalent static test set-up are shown in figure 10. The used set-up was statically determined. The resulting evolution of the stiffness value with the number of included modes is shown in figures 11, 12 and 13. The static bending stiffness calculated using the statics from dynamics method differed less than 0.7% from the stiffness calculated by means of an equivalent full static FE calculation, proving the capability of the statics from dynamics method for body-in-white static stiffness estimation.

From figures 11, 12 and 13, it can be concluded that certain modes have more influence than others; Others have no influence at all. In this test case, residual vectors were found to be of no influence on the static

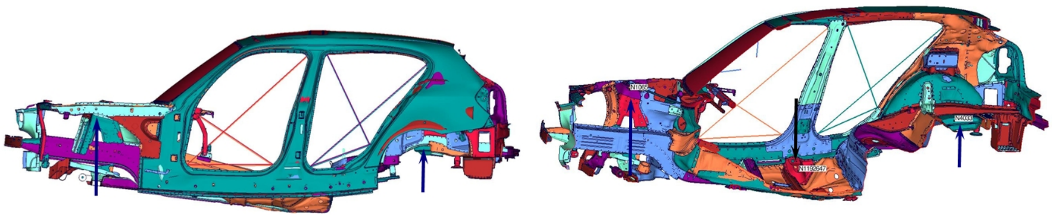

Figure 10: Force and support locations of equivalent static test set-up

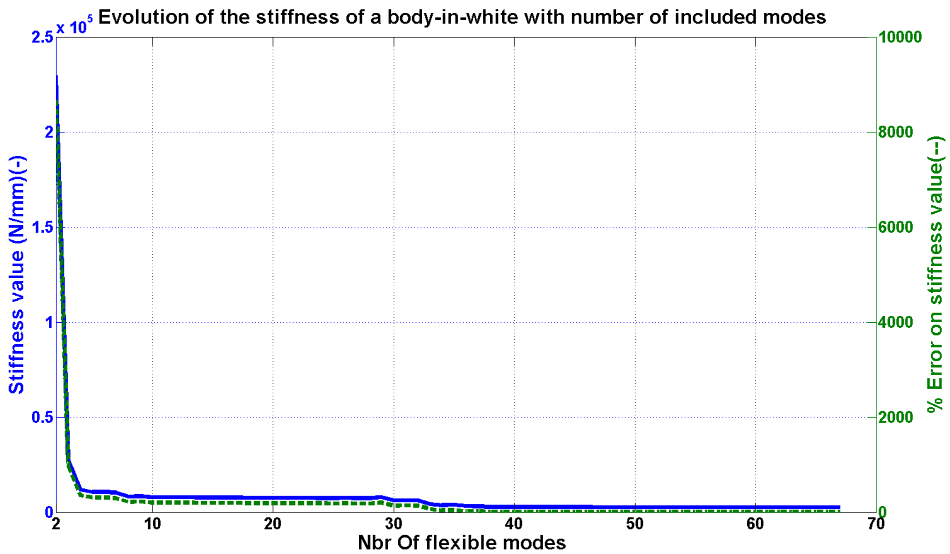

Figure 11: Evolution of the bending stiffness value with the number of modes taken into account (-) and the evolution of the error of the stiffness value with respect to the real value (- -).

stiffness value, implying that outer frequency band modes were not of influence anymore on the simulated modal set. This is mainly due to the large number of flexible modes used for the modal synthesis. From figures 11, 12 and 13 it is clear that influence of the dynamic modes on the static stiffness value decreases with the number of modes taken into account in the modal synthesis. Therefore the influence of the residual vectors decreases with the number of dynamic modes taken into account.

5.3 Modal contribution of flexible modes to global static stiffness

The connection between both the static and the dynamic stiffness can best be shown on basis of figures 11, 12 and 13. These figures define connection between the static bending stiffness, calculated with the statics from dynamics method, and the number of modes taken into account during calculation. There can be concluded that:

- One can clearly see, that some modes contribute more significantly to the static stiffness than others, resulting in large jumps in the static stiffness value.
- Moreover the influence of different modes decreases with increasing frequency, since the higher the eigenfrequency of the mode, the more local is the deformation pattern in the corresponding mode shape. This is of major importance for the statics from dynamics method because it implies that after

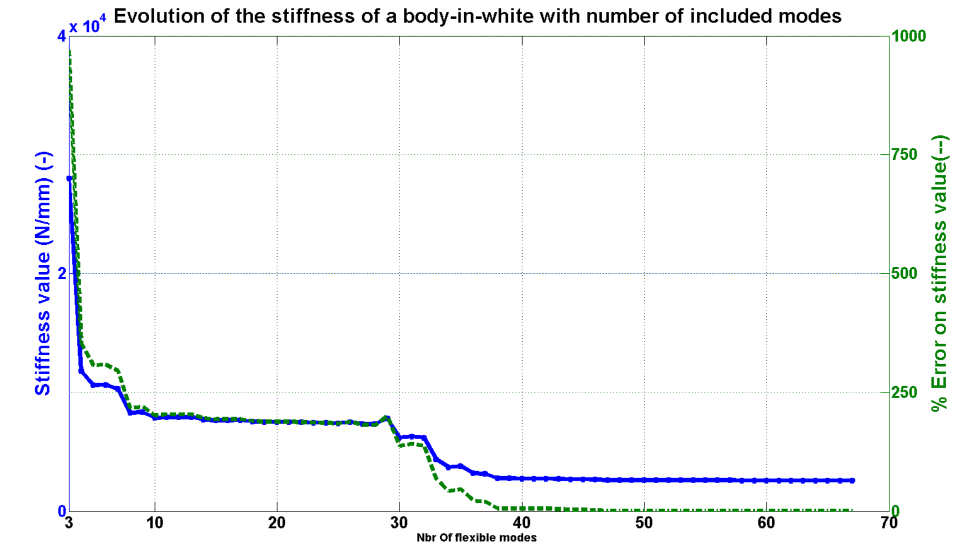

Figure 12: Evolution of the bending stiffness value with the number of modes taken into account (-) and the evolution of the error of the stiffness value with respect to the real value (- -).

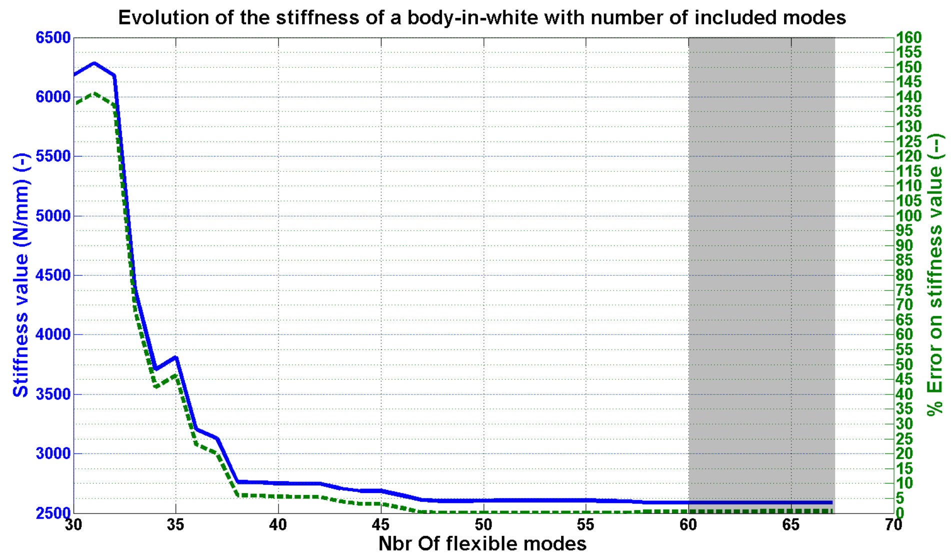

Figure 13: Evolution of the bending stiffness value with the number of modes taken into account (-) and the evolution of the error of the stiffness value with respect to the real value (- -). Residual vectors indicated by grey zone

a certain point it is no longer useful to take more modes into account. At this point the stiffness value has stabilized.

- Furthermore there is a link between the deformation pattern of the mode shape and the applied load case. This implies that, for example for a bending load case, the flexible modes that are of importance, will have a bending component in them. In other words, the static bending stiffness will be constructed using all flexible bending type modes.

Similar conclusions are valid for the static torsion stiffness.

6 Application of the method on experimental data[19]

The goal of this project was the development of a method to determine the global static stiffness of a body-in-white based on a single experimental model analysis test. To prove the experimental potential of the statics from dynamics method not only simulations, but also experimental tests were performed. The body-in-white testing was performed by Tuijtelaars and van der Tas [17]. In the current stage of the project, the direct experimental determination of compliance matrix approach was used. Hammer excitation delivered all elements of the FRF matrix. Curve fitting determined the compliance matrix at low frequency. Testing consisted of the body-in-white of three vehicle types: a SUV, a convertible and a limousine. First a comparison will be made between torsional stiffness values of the convertible resulting from on the one hand a FE based static torsional calculation and on the other hand from the statics from dynamics approach. Secondly results of a classic static torsion test of a SUV will be compared to those found using the statics from dynamics method.

6.1 Convertible

First the torsional stiffness comparison for a convertible is discussed. Reference will be a detailed FE calculation, in which clamping conditions are set at the rear and a torsion moment applied to the front. Global static torsional stiffness was derived from these calculations. Hammer measurements were performed between the base points. Reciprocity was kept in mind to keep the measurement effort to a minimum. One base point was used as driving point, whereas the cross-FRFs between this driving point and the other base points resulted from acceleration measurements.Compliance results between 8 and 20Hz were used in a curve fitting to determine the compliance at 0Hz. This compliance matrix was the input for the statics from dynamics method. Resulting stiffness values are listed in table 1. As expected from the simulation test, the statics from dynamics method overestimated the global stiffness value. Based on the simulation results it is clear that the static stiffness value found using the statics from dynamics method decreases with the number of modes taken into account. The higher in frequency the mode the lower its contribution to the overall static stiffness. Therefore, since it is expected to be impossible to estimate all dynamic modes from measurements performed in a limited frequency band and residual compensation will not be fully perfect, it was expected that not all modal information is taken into account, which results in an overestimation of the static stiffness value. Nevertheless the difference between the estimated value using the statics from dynamics method and the value resulting from the static reference test stays well below the five percent target.

6.2 SUV

For the SUV, similar experimental approach was used. However in this case the reference was a classic static test. The resulting stiffness values, listed in table 1, indicate better agreement between results than the five percent target. These two tests prove the practical validity of the statics from dynamics method.

Body set-up	Used Reference	Reference torsional stiffness	Statics from dynamics torsional stiffness	Difference (%)
Convertible	Finite Element Calculation	$21170 Nm/\circ$	$22017 Nm/\circ$	4.0
SUV	Classic Static Test	$31100 Nm/\circ$	$31165 Nm/\circ$	2.8

Table 1: Experimental benchmark results

7 Conclusions

This work describes the statics from dynamics method to determine global static stiffness of a structure. Two approaches have been discussed: the modal synthesis based compliance matrix estimation and the full experimental determination of the compliance matrix. The essence of the first approach is to accurately determine the modal model of a structure. Main advantage of the approach is the possibility to assess modal contribution of each mode to overall static stiffness. FE-simulation showed the potential of the method. However due to the complexity of residual vector estimation, especially in experiments, a second approach: The full experimental determination of the compliance matrix was implemented. Whereas this second approach requires more measurement effort, no residual vector estimation is necessary. The validity of the approach was proven based on experimental tests in body in white structures.

The statics from dynamics method is therefore a good alternative for classic static testing overcoming the reproducibility issues related to the clamping conditions in the classic static test and its time consuming set-up times.

In conclusion, the statics from dynamics method has the potential to become a good alternative for present classic static testing.

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